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UIMO
Unified International Mathematics Olympiad

UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

CLASS - 10
Question Paper Code : UM 9274

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EXPLANATIONS

MATHEMATICS

1. (D) Let the two sides of the triangle be x and y, then

$41^2 = x^2 + y^2 \Rightarrow x^2 + y^2 = 1681$

and $\frac{1}{2}xy = 180 \Rightarrow xy = 360$

$(x + y)^2 = x^2 + y^2 + 2xy = 1681 + 720 = 2401$

$\Rightarrow$ $x + y = 49 \Rightarrow x - y = \sqrt{(x+y)^2 - 4xy}$

$\Rightarrow$ $x - y = 31$ which is the required difference.

2. (B) Given equations are

$2x + 3y = 5$ ..... (1)

and $x - y = 10$ ..... (2)

Multiplying eq. (2) by 3 and adding eq. (1) and eq. (2), we get

$5x = 35$

$\Rightarrow x = 7$

and $y = -3$.

$\therefore$ The point (x, y) at which the submarine can be destroyed is (7, – 3).

3. (C) $(\sin x - \cos x)^4 = [(\sin x - \cos x)^2]^2$

$= (\sin^2 x + \cos^2 x - 2\sin x\cos x)^2$

$= (1 - 2\sin x\cos x)^2$

$= 1 + 4\sin^2 x\cos^2 x - 4\sin x\cos x$

$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x$

$= 1 + 2\sin x\cos x$

$\sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$

$= (\sin^2 x + \cos^2 x)^3$

$- 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)$

$= 1^3 - 3\sin^2 x\cos^2 x(1)$

$= 1 - 3\sin^2 x\cos^2 x$

$\therefore$ LHS $= 3(1 + 4\sin^2 x\cos^2 x - 4\sin x\cos x)$

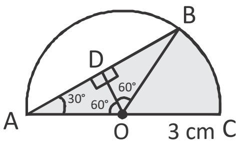
$+ 6(1 + 2\sin x\cos x) + 4(1 - 3\sin^2 x\cos^2 x)$

$= 3 + 12\sin^2 x\cos^2 x - 12\sin x\cos x$

$+ 6 + 12\sin x\cos x + 4 - 12\sin^2 x\cos^2 x$

$= 13$

4. (C)



Construction:-

Join OB and OD $\perp$ AB

In ΔAOD, $\sin 30° = \dfrac{OD}{AO}$

$\dfrac{1}{2} = \dfrac{OD}{3\,cm}$

$OD = \dfrac{3}{2}\,cm$

$\cos 30° = \dfrac{AD}{AO}$

$\dfrac{\sqrt{3}}{2} = \dfrac{AD}{3\,cm}$

$AD = \dfrac{3\sqrt{3}\,cm}{2}$

AB = 2 × AD

$= 2 \times \dfrac{3\sqrt{3}}{2}\,cm$

Area of the shaded region

= Area of ΔAOB + area of the sector BOC

$= \dfrac{1}{2} \times AB \times OD + \dfrac{x}{360°} \times \pi r^2$

$= \dfrac{1}{2} 3\sqrt{3}\,cm \times \dfrac{3}{2}\,cm + \dfrac{60°}{360°} \times 3.14 \times 3 \times 3\,cm^2$

$= \dfrac{9 \times 1.73}{4}\,cm^2 + \dfrac{1}{6} \times 3.14 \times 3 \times 3\,cm^2$

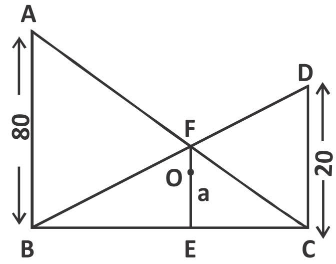
$= \dfrac{15.57}{4}\,cm^2 + 4.71\,cm^2$

$= 3.8958\,cm^2 + 4.71\,cm^2$

$= 8.6025\,cm^2$

5. (C) Δ ABC $\cong$ Δ DCB,

$\therefore \dfrac{AF}{CF} = \dfrac{AB}{CD} = \dfrac{80}{20} = \dfrac{4}{1}$



$\Rightarrow \dfrac{AF}{CF} = \dfrac{4}{1}$

$\Rightarrow \dfrac{AF + FC}{FC} = \dfrac{4+1}{1} = \dfrac{5}{1}$

ΔABC ~ ΔFEC [Since AB and FE are parallel]

$\Rightarrow \dfrac{AB}{FE} = \dfrac{AC}{FC}$

$\Rightarrow \dfrac{80}{FE} = \dfrac{5}{1}$

FE = 16

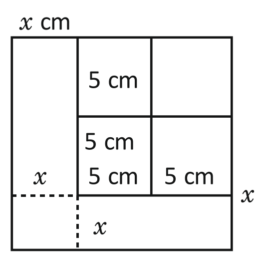
Hence, the value of a = 16 m

6. (A) Area of each same square $= \dfrac{125\,cm^2}{5}$

$= 25\,cm^2$

$\therefore$ Side of each small square = 5 cm

let the smallest side be x cm



$\therefore$ Area of 'L' shaped part $= 10x + x^2 + 10x$

$= x^2 + 20x$

$\therefore$ Given $x^2 + 20x = 25$

$x^2 + 20x - 25 = 0$

$a = 1, b = 20, c = -25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-20 \pm \sqrt{20^2 - (-204C_1)}}{2 \times 1}$$

$$= \frac{-20 \pm \sqrt{400 + 100}}{2}$$

$$x = \frac{-20 \pm \sqrt{500}}{2}$$

$$= \frac{-20 \pm 10\sqrt{5}}{2}$$

$$x = \frac{-20 + 10\sqrt{5}}{2} = \cancel{10}^{5}(-2 + \sqrt{5})$$

$$= 5(\sqrt{5} - 2) \text{ cm}$$

7. (A) Given $\frac{x}{1} = \frac{\sqrt{a+3b} + \sqrt{a-3b}}{\sqrt{a+3b} - \sqrt{a-3b}}$

$\therefore$ $$\frac{x+1}{x-1} = \frac{\left(\sqrt{a+3b} + \sqrt{a-3b}\right) + \left(\sqrt{a+3b} - \sqrt{a-3b}\right)}{\left(\sqrt{a+3b} + \sqrt{a-3b}\right) - \left(\sqrt{a+3b} - \sqrt{a-3b}\right)}$$

$$\frac{x+1}{x-1} = \frac{2\sqrt{a+3b}}{2\sqrt{a-3b}}$$

Squaring on both sides.

$$\frac{x^2 + 2x + 1}{x^2 - 2x + 1} = \frac{a + 3b}{a - 3b}$$

$$\cancel{ax^2} + 2ax + \cancel{a} - 3bx^2 - \cancel{6bx} - 3b = \cancel{ax^2} - 2ax + \cancel{a} + 3bx^2 - \cancel{6bx} + 3b$$

$0 = 6bx^2 - 4ax + 6b$

$\therefore$ $3bx^2 - 2ax + 3b = 0$

8. (D) Let the three sides of a right angled triangle be a, a + d, a + 2d respectively [$\because$ Given sides are in AP]

$\therefore$ $(a + 2d)^2 = a^2 + (a + d)^2$

$$\cancel{a^2} + 4ad + 4d^2 = \cancel{a^2} + a^2 + 2ad + d^2$$

$\Rightarrow a^2 - 2ad - 3d^2 = 0$

$a^2 - 3ad + ad - 3d^2 = 0$

$a(a - 3d) + d(a - 3d) = 0$

$\Rightarrow (a - 3d)(a + d) = 0$

$a - 3d = 0$ or $a + d = 0$

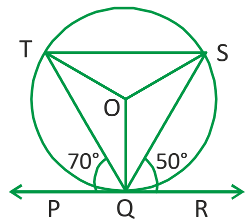
$\therefore$ a = 3d & a = –d rejected because side of triangle is always positive

$\therefore$ 3d, 4d and 5d are the sides of a right angled triangle

$\therefore$ 80 Unit is the side of a right angled triangle because 80 is multiple of 4 as well as 5

9. (C) $\angle OQS = 90° - 50° = 40°$

$\therefore \angle OSQ = \angle OQS = 40°$



$\therefore \angle OQS = 180° - 40° - 40° = 100°$

$$\therefore \angle QTS = \frac{\angle OQS}{2} = 50°$$

$\angle TQP = 70°$

[$\because$ Given] $\Rightarrow \angle TQO = 20°$

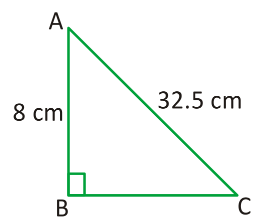
$\therefore$ $\angle OTQ = \angle TQO = 20°$

$\Rightarrow \angle OTS = \angle QTS - \angle OTQ = 50° - 20°$

$\therefore$ $\angle OTS = \angle x = 30°$ [$\because$ OT = OS]

10. (D) In ΔABC given $\angle B = 90°$

$\therefore$ $AC^2 = AB^2 + BC^2$



$(32.5) = 8^2 + BC^2$

$$BC = \sqrt{1056.25 - 64} = \sqrt{992.25} = 31.5$$

Area of DABC

$$= \frac{1}{2} \times AB \times BC = \frac{1}{\cancel{2}_1} \times \cancel{8}^4 \text{ cm} \times 31.5\text{cm}$$

$= 126 \text{ cm}^2$

11. (D) The required distance

$$= \sqrt{\left[\left(\sqrt{3}+1\right)-\left(\sqrt{3}-1\right)\right]^2+\left[\left(\sqrt{2}-1\right)-\left(\sqrt{2}+1\right)\right]^2}$$

$$= \sqrt{(2)^2+(2)^2} = 2\sqrt{2}$$

12. (D) Area of the semicircle becomes curved surface area of the cone and radius of the semicircle becomes slant height of cone

$\therefore \quad \pi r l = \frac{1}{2}\pi \times (2\sqrt{3}\text{ cm})^2$

$r(2\sqrt{3}) = \frac{1}{2} \times 2\sqrt{3} \times 2\sqrt{3}$ cm

$r = \sqrt{3}$ cm

$h = \sqrt{l^2 - r^2} = \sqrt{(2\sqrt{3})^2 - (\sqrt{3})^2} = 3$ cm

Volume of the cone =

$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (\sqrt{3})^2 \times 3$ cm

$= 3\pi$ cm^2

13. (D) Let the smallest integer be x.

Given $\frac{30}{2}(x + x + 29) = 315$

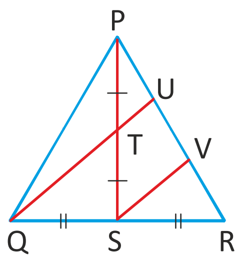
$2x + 29 = \frac{315}{15} = 21$

$2x = 21 - 29 = -8$

$x = -4$

14. (C) Const: Draw SV||QU

In Δ QRU, S is the mid point of QR and SV || QU



$\therefore \quad \frac{RS}{SQ} = \frac{RV}{VU}$ [$\because$ Thales theorem]

$\therefore \quad \frac{RS}{RS} = \frac{RV}{VU}$

$\Rightarrow$ RV = VU ⟶ ①

In Δ PSV, TU||SV

$\Rightarrow \frac{PT}{PS} = \frac{PU}{UV}$ [$\because$ Thales theorem]

$\Rightarrow$ PU = UV ⟶ ②

from ① & ②, PU = UV = VR

$\therefore \quad PU = \frac{1}{3}PR = \frac{1}{3} \times 24$ cm $= 8$ cm

15. (A) (A) Let $a\sin\theta + b\cos\theta = y$

$x^2 + y^2 = (a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2$

$= a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta$

$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$= a^2 + b^2$

$\therefore \quad y^2 = a^2 + b^2 - x^2$

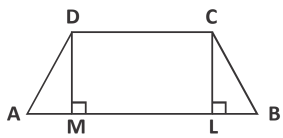
$y = \pm\sqrt{a^2 + b^2 - x^2}$

16. (A) $S = \frac{P}{2} = 12$ cm

Area of $\Delta = rs = 12$ cm $\times$ 2 cm $= 24$ cm^2

17. (B) From C, draw CL $\perp$ AB and from D, drawn DM $\perp$ AB

Then CL = DM



In ΔACB, since $\angle$B is an acute angle,

$\therefore$ AC^2 = AB^2 + BC^2 – 2AB . BL .... (1)

Similarly, In ΔABD, Since $\angle$A is an acute angle,

$\therefore$ BD^2 = AD^2 + AB^2 – 2AB . AM ..... (2)

Adding (1) and (2), we get

$AC^2 + BD^2 = AD^2 + BC^2 + 2AB^2 - 2AB \cdot BL - 2AB \cdot AM$

$= AD^2 + BC^2 + 2AB\,(AB - BL - AM)$

$= AD^2 + BC^2 + 2AB\,(AL - AM)$

$= AD^2 + BC^2 + 2AB \cdot ML$

$= AD^2 + BC^2 + 2AB \cdot CD$

18. (D) $\alpha = a - d,\ \beta = a,\ \gamma = a + d$

Given α, β, γ are in AP

$\Rightarrow\ \alpha + \beta + \gamma = -\frac{b}{a}$

$a - d + a + a + d = 9$

$3a = 9$

$a = 3$

$\alpha\beta\gamma = 21$

$(a - d)(a)(a + d) = -21$

$(3 - d)(3)(3 + d) = -21$

$(3 - d)(3 + d) = -7$

$9 - d^2 = -7$

$d^2 = 16$

$d = \pm 4$

If $a = 3$ & $d = 4$ then $a - d = -1$, $a + d = 7$

If $a = 3$ & $d = -4$ then $a - d = 7$, $a + d = -1$

$\therefore\ a + d - (a - d) = 2d = 8$

19. (D) $\cos 30° \cos 45° - \sin 30° \sin 45°$

$= \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4}$

20. (A) If $n = 0$ then $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{a+b}{2}$ which is Am of a & b.

21. (C) Given vertices form a right angled triangle In a right angled triangle the vertex of right angle is orthocentre.

$\therefore$ Ortho centre = (0, 0)

22. (D) $BD = 2\sqrt{2}$

$\therefore r = \frac{BD}{2} = \frac{2\sqrt{2}}{2}\text{ cm} = \sqrt{2}\text{ cm}$

Area of Circle = $\pi\left(\sqrt{2}\right)^2 = 2\pi\ \text{cm}^2$

Area of square $= 2^2 = 4$

Area of circle – Area of square = $2\pi - 4$

Area between circle and square on each side = $\frac{2\pi - 4}{4}$

Area of shaded regions

$= 4\left[\frac{\pi(1)^2}{2} - \frac{\pi - 2}{2}\right]$

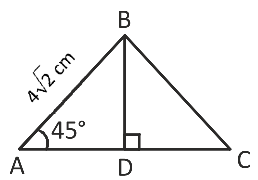
$= 4\left[\frac{\pi - \pi + 2}{2}\right] = 4\ \text{cm}^2$

23. (C) HCF of $\frac{5}{12}, \frac{10}{9}, \frac{25}{6}$ is $\frac{5}{36}$

LCM of $\frac{5}{12}, \frac{10}{9}, \frac{25}{6}$ is $\frac{50}{3}$

$\therefore$ LCM + HCF = $\frac{5}{36} + \frac{50}{3} = \frac{5+600}{36} = \frac{605}{36}$

24. (B) Const:- $BD \perp AC$



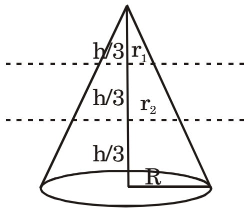
In ΔABD $\sin 45° = \frac{BD}{AB}$

$\frac{1}{\sqrt{2}} = \frac{BD}{4\sqrt{2}\text{ cm}}$

BD = 4 cm

Area of ΔABC = $\frac{1}{2} \times BD \times AC = \frac{1}{2} \times 4$ cm × 7 cm = 14 cm^2

25. (D)



$$v_1 : v_2 : v_3 = \frac{1}{3}\pi\left(\frac{r}{3}\right)^2\left(\frac{h}{3}\right)$$

$$:\left[\frac{1}{3}\pi\left(\frac{2r}{3}\right)^2\left(\frac{2h}{3}\right) - \frac{1}{3}\pi\left(\frac{r}{3}\right)^2\left(\frac{h}{3}\right)\right]:$$

$$\left[\frac{1}{3}\pi r^2 h - \frac{1}{3}\pi\left(\frac{2r}{3}\right)^2\left(\frac{2h}{3}\right)\right]$$

$$= \frac{\pi r^2 h}{81} : \frac{7\pi r^2 h}{81} : \frac{19\pi r^2 h}{81}$$

= 1 : 7 : 19

26. (A) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$

They are dependent equations

27. (B) Given $\angle ACP = a°$ and $\angle BPC = b°$.

from the figure, $\angle APC = \angle APB + \angle BPC$

but $\angle APB = 90°$

($\because$ angle in a same - circle)

$\therefore \angle APC = 90° + b°$

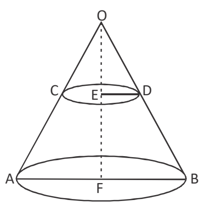
now in ΔAPC, we have $\angle ACP = a°$ and $\angle APC = 90° + b°$ and $\angle CAP = b°$
($\because \angle BPC = \angle CAP$ alternate angles)

$\therefore$ sum of 3 angles of ΔAPC

a° + 90° + b° + b° = 180°

a° + 2b° = 90°

28. (A) Let OAB be the given cone cut off by a plane CD parallel to the base AB such that a small cone OCD is left.



For the cone OAB :

Height H = 30 cm

Let the radius of the base be R.

Then, Volume of the cone OAB

$$= \frac{1}{3}\pi R^2 h$$

$$= \left[\frac{1}{3}\pi R^2 \times 30\right] cm^2 = \left(10\pi R^2\right) cm^3$$

For the cone OCD :

Let the height be h and radius of the base be r.

Then, the volume of the cone OCD

$$= \frac{1}{3}\pi R^2 h.$$

New, $\frac{1}{3}\pi r^2 h = \frac{1}{27}\left(10\pi R^2\right)$

[given]

$$\Rightarrow \left(\frac{R}{r}\right)^2 = \frac{9h}{10} \quad \text{... (i)}$$

Also, $\Delta OED \sim \Delta OFB$ [$\because$ OF = H = 30 cm]

$$\therefore \frac{OE}{OF} = \frac{ED}{FB} \Rightarrow \frac{h}{30} = \frac{r}{R} \Rightarrow \frac{R}{r} = \frac{30}{h} \quad \text{... (ii)}$$

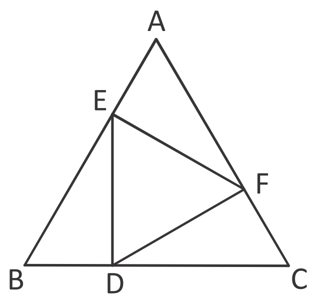
From (i) and (ii) we get :

$$\left(\frac{30}{h}\right)^2 = \frac{9h}{10} \Rightarrow h^3 = 1000 \Rightarrow h = 10 \text{ cm.}$$

Thus, the height of the smaller cone OCD = 10 cm.

Hence, the height of the section from the base = EF = OF – OE = H – h = (30 – 10) cm = 20 cm.

29. (C)



Let's take one of the smaller right triangles. Without loss of generality, let the smaller leg be 1. Since the triangle is a 30° – 60° – 90° right triangle, then the other leg is $\sqrt{3}$ and the hypotenuse is 2. The side length of the bigger triangle is 1 + 2 = 3 and the side length of the smaller triangle is $\sqrt{3}$. The ratio of the areas of two similar triangles is the square of the ratio of two corresponding side lengths, so the ratio of the are of triangle DEF to the area of triangle ABC is $\left(\frac{\sqrt{3}}{3}\right)^2 = \frac{1}{3}$.

30. (B) Circumferce of circular path

$= 2\pi r = 2 \times \frac{22}{7} \times 14$ m

= 88 m

Time taken to cover 88 m to Narayana

$= \frac{88\text{ m}}{17.6\text{ KMPH}} = \frac{88\text{ m}}{17.6 \times \frac{5\text{ m}}{18\text{ sec}}}$

= 18 seconds

Time taken to cover 88 m to krishna

$= \frac{88\text{ m}}{26.4\text{ KMPH}} = \frac{88\text{ m}}{26.4 \times \frac{5\text{ m}}{18\text{ sec}}}$

= 12 seconds

LCM of 18 seconds and 12 seconds is 36 seconds.

31. (B, C, D) Mid pt. of AB = (0, 2) ; length of median to AB $= \sqrt{26}$

Mid pt. of BC = (3, 1) ; length of median to BC = $2\sqrt{5}$

Mid pt. of AC = (2, 2); length of median to AC = $\sqrt{2}$

Hence, option (A) is not the length of median.

32. (B, D) If $\theta = 30°$ then

$81^{\sin^2\theta} + 81^{\text{con}^2\theta}$

$= 81^{\left(\frac{1}{2}\right)^2} + 81^{\left(\frac{\sqrt{3}}{2}\right)^2}$

$= 81^{\frac{1}{4}} + 81^{\frac{3}{4}}$

$= \left(3^4\right)^{\frac{1}{4}} + \left(3^4\right)^{\frac{3}{4}}$

= 3 + 3^3

= 3 + 27 = 30

Similarly $\theta = 60°$ alos

$81^{\sin^2\theta} + 81^{\text{con}^2\theta} = 30$

33. (B, C, D) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ consistent but unique solution

If $\frac{a_1}{a_2} = \frac{c_1}{c_2}$ and either

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ or $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$

then the given lines are consistent

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ the lines are consistent

34. (A, C) $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

$\sqrt{3}\cot^2\theta - 3\cot\theta - \cot\theta + \sqrt{3} = 0$

$\sqrt{3}\cot\theta\left(\cot\theta - \sqrt{3}\right) - 1\left(\cot\theta - \sqrt{3}\right) = 0$

$\left(\cot\theta - \sqrt{3}\right)\left(\sqrt{3}\cot\theta - 1\right) = 0$

$\cot\theta - \sqrt{3} = 0$ (or) $\sqrt{3}\cot\theta - 1 = 0$

$\cot\theta = \sqrt{3}$

$\cot\theta = \cot 30°$

$\theta = 30°$

$\sqrt{3}\cot\theta = 1$

$\cot\theta = \frac{1}{\sqrt{3}} = \cot 60°$

$\theta = 60°$

35. (A, B, C, D)

Let the three angles of a triangle are in AP be

a – d, a, a + d

∴ a – d + a + a + d = 180°

3a = 180°

a = 60°

∴ If d = 10° then

50°, 60° and 70° are in AP and angles of a triangle

If d = 30° then 30°, 60° and 90° are in A

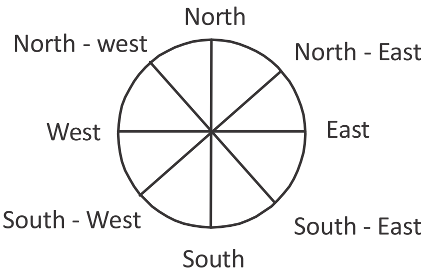
If d = 2° then

58°, 60° and 62° are in AP

If d = 0° then 60°, 60° and 60° are also in AP.

36. (A) Z - V; C - B; X - N shows pairs opposite faces.

37. (C)



If South - East becomes North and North East becomes West, therefore, the whole figure moves throught 135°. Hence, West will be South - East. See, Actual figure is rotating 135° anticlockwise. So When West will be rotated by same degree anticlockwise. It will hold the place of south - East.

38. (A) $13^{th} \longrightarrow 5^{th}$

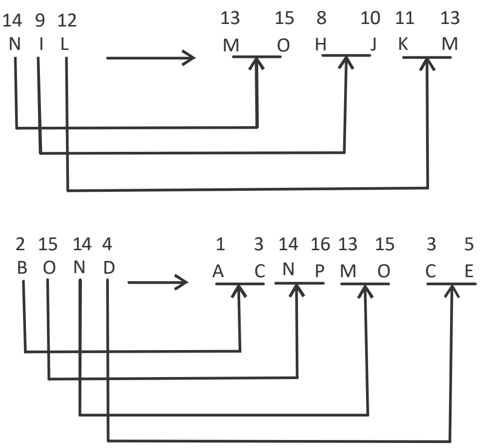
A B C D **E** F G H I J K L M N O P **Q** R S

39. (C) (1 + 9 + 2 + 7) × 2

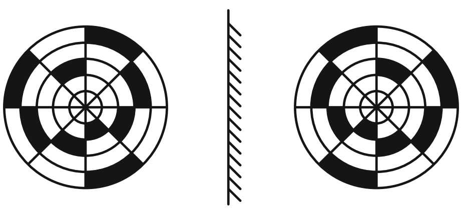
19 × 2 = 38.

40. (A) In all the other figures, both pin and arrow are in opposite directions and parallel to each other.

41. (C)



42. (B)



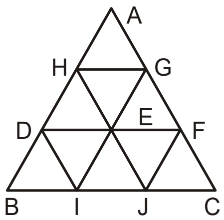
43. (C) The short arrow rotates 45° ACW and long arrow rotates 90° CW.

Hence, option (D), should replace the question mark.

44. (A) 4 → 5 → 9 → 18 → 34 → 59

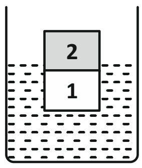
$+1^2$, $+2^2$, $+3^2$, $+4^2$, $+5^2$

45. (C) There are 15 parallelograms in the given figure.



1. BDEI 2. EFJI3. DEJI 4. EFCJ 5.DEGH 6. EFGH 7. EGAH 8. BHGI 9. GHJC 10. AGID 11. AFJH12. DHEI 13. EGFI 14. DFJB 15. DFCI

46. (B)



47. (D) To solve this question, you need to know how to set up a system of equations using the given information and how to solve for the unknown variable. Here is one possible explanation:

Let x be the number of bottles that Har fills in one minute and y be the number of bottles that Shiva fills in one minute. Then, we have the following equations.

$x + y = 30$ (the total number of bottles filled by Hari and Shiva in one minute $\frac{900}{30}$)

Statement 1 tells us that $y = x/2$, which means that Shiva fills half as many bottles as Hari in one minute. We can substitute this into the first equation and get:

$x + x/2 = 30$

$3x/2 = 30$

$x = 20$

This means that Hari fills 20 bottles in one minute and Shiva fills 10 bottles in one minute. We can use this to find how long it takes Shiva to fill the bottles by herself.

$30y = 900$

$30(10) = 900$

$y = 30$

So, Shiva takes 30 minutes by herself to fill the bottles. Therefore, statement 1 alone is sufficient to answer the question.

Statement 2 tells us that $45x = 900$, which means that Hari would take 45 minutes by himself to fill the bottles. We can solve for x and get:

$x = 20$

This is the same as what we found from statement 1, so we can use it to find how long it takes Shiva to fill the bottles by herself:

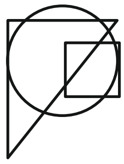
$y = x/2$

$y = 20/2$

$y = 10$

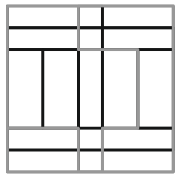
So, Shiva fills 10 bottles in one minute and takes 30 minutes by herself to fill the bottles. Therefore, statement 2 alone is also sufficient to answer the question.

Since both statements alone are sufficient, the correct answer is D. Each statement alone is sufficient.

48. (B) 

The dot should become under three region.

49. (B) Three of the statements are correct, and only one digit is on the card. Thus, one of I and III are false. Therefore, II and IV must be true.

50. (D) 

The End